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# HEAT TRANSFER TO FALLING LIQUID FILMS AND FILM BREAKDOWN—I

## SUBCOOLED LIQUID FILMS

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**Abstract**—Experiments are described on heat-transfer coefficient and film breakdown heat flux for subcooled water films flowing downwards on outside of a vertical tube uniformly heated. A distortion parameter of liquid films, which is a ratio of an allowable difference in surface tension to the dynamic pressure of uniform film flow, is derived for each laminar and turbulent film flow. Experimental data show that when the parameter attains a constant value by increasing heat flux, the film breakdown takes place in a thin region of the distorted film. A simple analysis is also presented for evaluating film distortion.

### NOMENCLATURE

$c$, specific heat;
$g$, acceleration of gravity;
$h$, heat-transfer coefficient, $h = q_o/(t_o - t_f)$;
$h^*$, dimensionless heat-transfer coefficient, $h^* = \frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3}$;
$k$, thermal conductivity;
$K$, distortion parameter, $K = \Delta\sigma/P_d y_i$;
$L$, length of heating section;
$P_d$, dynamic pressure, $P_d = \int_0^{y_i} \rho u^2\,dy/2y_i$;
$Pr$, Prandtl number, $Pr = \mu c/k$;
$q$, heat flux;
$q_1$, heat flux at local dry patch formation;
$q_2$, heat flux at permanent dry patch condition;
$Re$, Reynolds number, $Re = 4\Gamma_f/\mu$;
$t, t_f$, temperature, bulk mean temperature of liquid film;
$u, u_f$, velocity, mean velocity of liquid film;
$We$, Weber number, $We = \int_0^{y_i} \rho u^2 dy/2\sigma = P_d y_i/\sigma$;
$x$, distance from upper end of heating section;
$y, y_i$, distance from wall surface, film thickness.

Greek symbols

$\Gamma_f$, mass flow rate per unit periphery;
$\mu$, viscosity;
$\nu$, kinematic viscosity;
$\rho$, density;
$\sigma$, surface tension.

Subscripts

$i$, film surface;
in, upper end of heating section, $x = 0$;
$l$, laminar film;
$o$, wall surface;
out, lower end of heating section, $x = L$;
$t$, turbulent film.

### 1. INTRODUCTION

HEAT transfer to liquid films flowing on a heating surface and breakdown of the films associated with increasing heat flux are of great importance in some chemical engineering operations and in cooling systems of nuclear fuel clusters. The film breakdown is also relevant to the critical heat flux of high quality boiling in channels. A number of investigations [1] have, therefore, been conducted on these subjects and it has been shown that film breakdown conditions are influenced by several different mechanisms. However, knowledge of the precise nature of film breakdown is still incomplete.

In this study, experiments were made on heat-transfer coefficient and film breakdown conditions—both conditions of local and permanent dry patch formations—for highly subcooled water films flowing downwards on outside of a vertical tube uniformly heated. Then, based on the results, the process was investigated of the film breakdown which was caused by the surface tension difference set up in the film.

The heating sections employed in this experiment were electrically heated stainless steel tubes of 600 and 1000 mm length. Measurements were made in a range of inlet water temperatures $t_{f\,in}$ from 22 to 80°C and film flow rates per unit periphery $\Gamma_f$ from 0.06 to 0.8 kg/m·s at the atmospheric pressure. The film Reynolds numbers $Re = 4\Gamma_f/\mu$ ranged from 500 to 5000.

The film flow rates under investigation are above the "minimum wetting rate", i.e. the minimum liquid flow rate required to wet the surface or to re-wet the surface after formation of a dry patch, for isothermal flow. And the heat fluxes applied in this experiment are lower than those at which film breakdown will occur by boiling. A study on the breakdown of boiling films will be presented in Part 2.

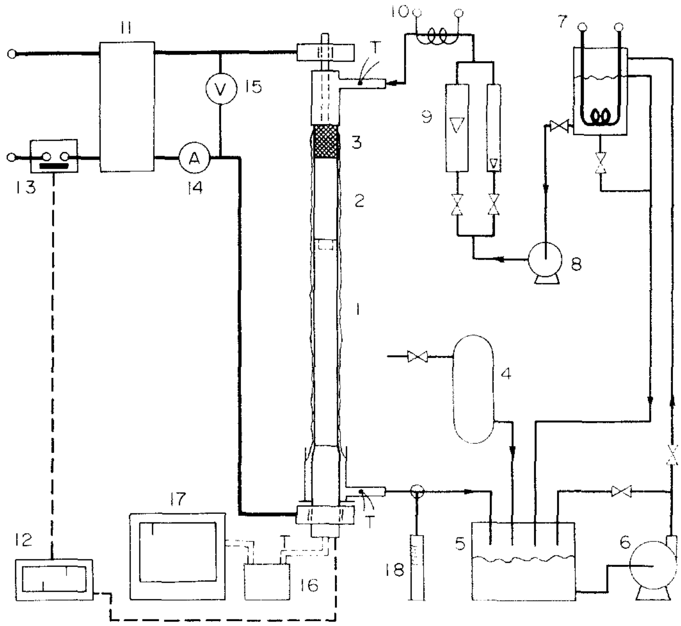


FIG. 1. Schematic diagram of experimental apparatus: 1, heating tube; 2, intake region; 3, sinter; 4, ion exchanger; 5, main tank; 6, pump; 7, preheater tank; 8, circulation pump; 9, rotameters; 10, control heater; 11, voltage adjuster and transformer; 12, 13, excess temperature trip; 14, current transformer and ammeter; 15, voltmeter; 16, ice box; 17, temperature recorder; 18, mess-cylinder; T, thermocouple.

## 2. EXPERIMENTAL EQUIPMENT AND PROCEDURE

A flow diagram of the apparatus is shown in Fig. 1. The test section arranged vertically consists of the water inlet section, intake region and heating section. The heating section is a stainless steel tube of 16 mm outside diameter, 1 mm thickness and 600 or 1000 mm length. A copper rod of the same diameter as the heating section and 250 mm length is brazed at upper end of the heating section as the intake region of film flow. At the water inlet section a porous sinter tube of 70 mm length is provided to form a circumferentially uniform liquid film. Water supplied passes through the sinter tube from inside to outside and flows down on the outside surface of the intake region and heating section.

The test section was used as an electrical resistance and was heated by applying a low voltage alternating current through copper connectors and copper conductors brazed on both ends of the heating section.

For measuring the wall temperatures of the heating tube, nine thermocouples supported by Teflon pieces on a steel axle were inserted from bottom of the tube and touched the inner surface of the tube.

Test procedure was as follows. The test section was pretreated by polishing and cleaning, and was wetted satisfactorily by an hour of water circulation. After the inlet water temperature and flow rate were adjusted to a set of values, power was supplied to the heating section and increased with a small increment until a permanent dry patch was observed or until an excess temperature trip set at 140°C operated to cut off the power. Heat flux was permitted up to $1.8 \times 10^5$ W/m$^2$ with a power supply used in this experiment.

## 3. FILM FLOW STATE

The film surface was wavy on the heating section, although it remained flat and smooth within a short distance from the sinter. With an increase of heat flux, the film configuration changed as illustrated in Fig. 2(a)–(d).

(a) At zero heat flux, a circumferentially uniform film with fine wave crests extending around the periphery flowed down the test tube without much visible change, although an increase of flow rate made the film surface a little more irregular as the film flowed down the tube.

(b) When a heat flux was applied, distortion of wave crests and local thinning of the film were observed. In this state, the water drawn away from the thinning part of the film was diverted to another part of the periphery where the film thickness and velocity were greater than the averages. The distortion of the film was promoted with increasing heat flux and became marked near the film breakdown heat flux.

(c) At the heat flux $q_1$, local dry patches were observed. These dry patches were usually formed in the thin film region downstream of a distorted crest, on down half of the heating section. In many cases, a large wave produced upstream by piling up the crests flowed

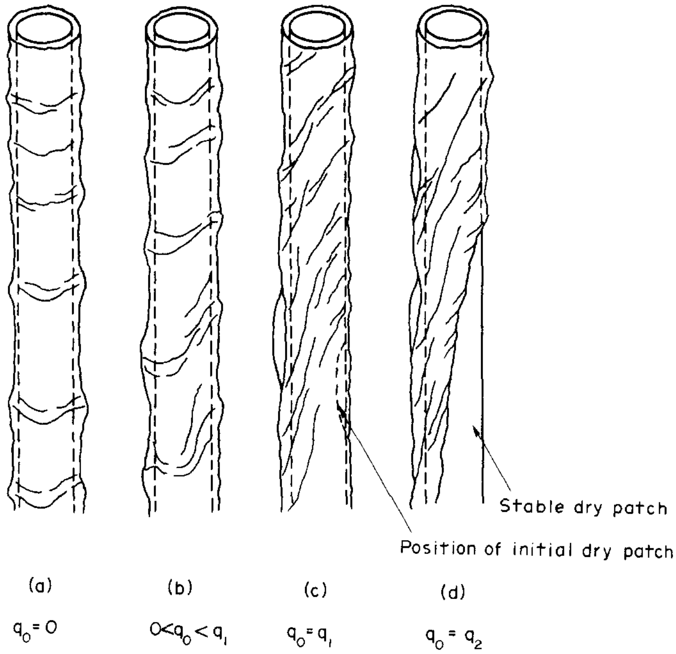


FIG. 2. Variation of film flow state with increasing heat flux.

down recovering the dry patch, then dry patch formation and re-wetting were repeated.

(d) The further increase of heat flux attenuated the wave motion upstream of the dry patch by extending the thin film region. This led to formation of a large stable dry patch at the heat flux $q_2$. At this stage, most of the water was diverted to the tube surface diametrically opposite the dry patch, and flowed down surrounding the dry patch in a shape of parabola. When the temperature of this dry surface was high enough to cause violent nucleate boiling, water was ejected by sputtering action at the interface of the dry patch.

Typical data of the wall temperature variation with time at film breakdown heat fluxes are shown in Fig. 3, which were measured by means of three thermocouples on the inner surface located at 440 mm downstream of the upper end of the heating section and with an equal peripheral interval. The wall temperature corresponding to a thin part of the liquid film showed a high value with fluctuation. At the heat flux $q_1$, formation of local dry patches and re-wetting showed a high peak of wall temperature. Formation of a permanent dry patch at the heat flux $q_2$ was always accompanied with a sharp rise of wall temperature, as shown in the RHS of this figure.

#### 4. HEAT-TRANSFER COEFFICIENT

For heat transfer to falling liquid films, many experimental results and several analytical solutions have been reported with relation to such cases as filmwise condensation and annular two phase flow.

In the present paper, local heat-transfer coefficient to a falling film and its dimensionless form are defined by

$$h = \frac{q_o}{t_o - t_f}, \quad h^* = \frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3} \tag{1}$$

where $t_o$ is the temperature of the wall surface and $t_f$ is the bulk mean temperature of the liquid film.

For a laminar falling film fully developed and with constant heat rate along the wall, analytical solutions [2, 3] are easily obtained by introducing the film thickness derived by Nusselt. In the case where all of the heat transferred from the heating surface are absorbed in the liquid film,

$$h^* = 2.27 Re^{-1/3} \tag{2}$$

and in another case where they are removed away from the film surface,

$$h^* = 1.76 Re^{-1/3}. \tag{3}$$

Experimental studies of heat-transfer coefficient to falling liquid films have been performed by Wilke [4] and Ishigai *et al.* [5] for subcooled liquids and by Chun and Seban [6] for a saturated liquid with evaporation.

Wilke has measured heat-transfer coefficient to a liquid film flowing downwards on outside of a 2400 mm length vertical tube heated by hot water flowing through inside of the tube. In his experiment, data were obtained over a wide range of liquid Prandtl numbers from $Pr = 5.4$ to 210, by using water/glycol mixtures of various concentrations. The empirical equations thus derived by Wilke can be transformed into relations between $h^*$ and $Re$ by introducing the film thickness derived by Nusselt for $Re < 1600$ and that given by Brauer [7] for $Re \geqslant 1600$. The resultant equations for water are expressed as

$$Re \leqslant 2460 Pr^{-0.646}; \quad h^* = 1.76 Re^{-1/3} \tag{4}$$

$$2460 Pr^{-0.646} \leqslant Re < 1600; \quad h^* = 0.0323 Re^{1/5} Pr^{0.344} \tag{5}$$

$$1600 \leqslant Re < 3200; \quad h^* = 0.00102 Re^{2/3} Pr^{0.344} \tag{6}$$

$$3200 \leqslant Re; \quad h^* = 0.00871 Re^{2/5} Pr^{0.344}. \tag{7}$$

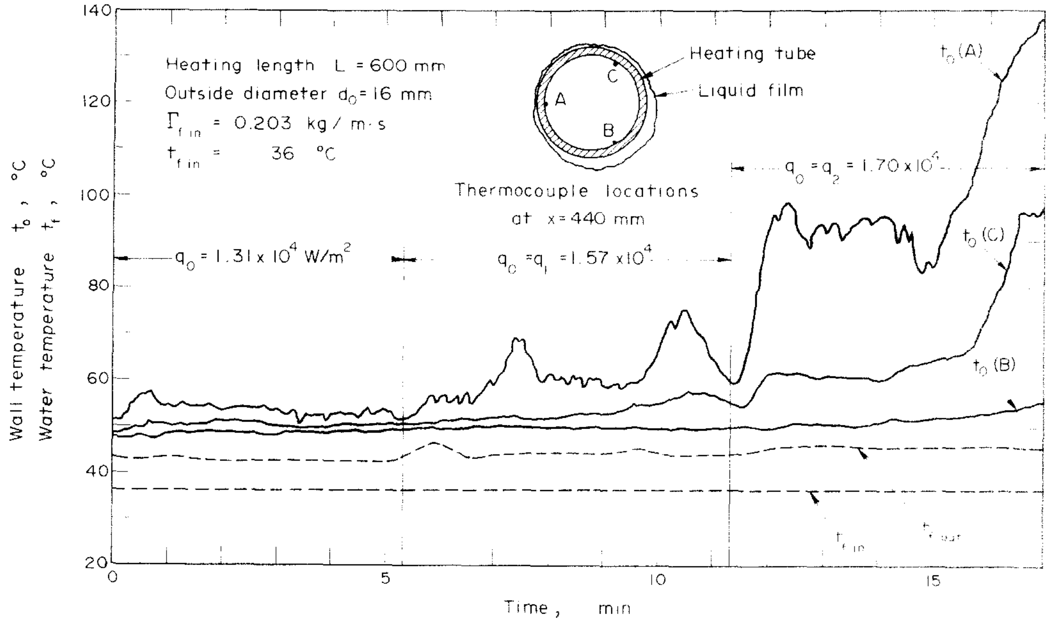


FIG. 3. Variation of wall temperatures at film breakdown heat fluxes.

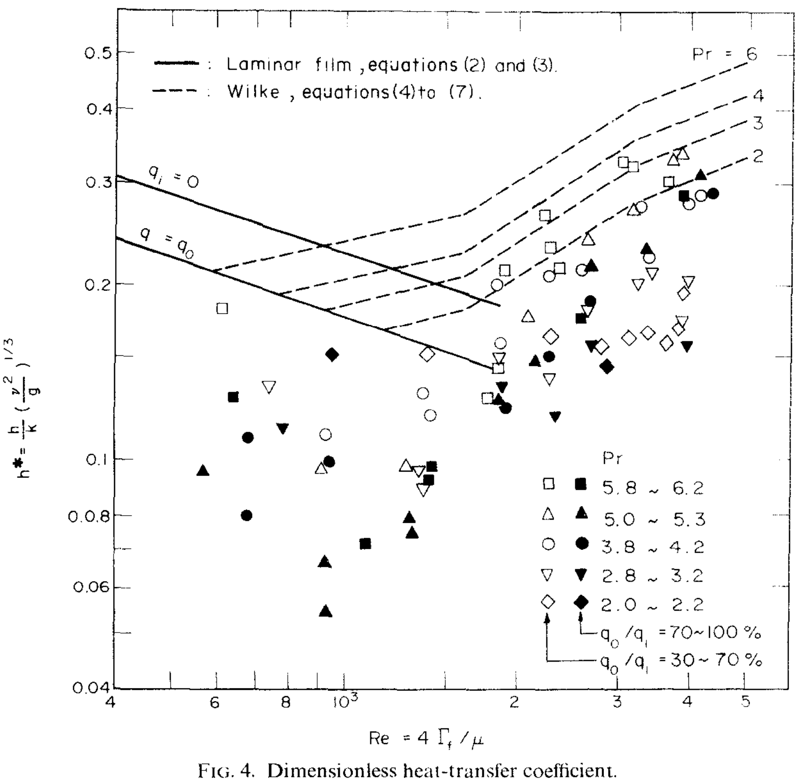


FIG. 4. Dimensionless heat-transfer coefficient.

Figure 4 shows the results at $x = 440$ mm of the present experiment. The heat-transfer coefficient was calculated based on an average of the temperatures at the outer wall surface which were obtained by the measured values at three locations along the inner periphery of the tube taking into account the temperature differences between both sides of the wall. The bulk mean temperature of the liquid film was determined by interpolating the liquid temperatures measured at both inlet and outlet of the heating section. In this experiment, heat removal from the film surface to the air was small in comparison with the heat flux $q_0$. In this figure, the experimental points are separated into two groups according to the measured heat flux to the film breakdown heat flux $q_1$.

Two solid lines in Fig. 4 represent the values of $h^*$ given by equations (2) and (3), and broken lines the values according to equations (4)–(7). In a low heat flux range, the relation between $h^*$ and $Re$ shows a trend similar to that presented by Wilke, but the measured values of $h^*$ are a little lower than Wilke's. In a high heat flux range, near the film breakdown heat flux $q_1$, the measured values are comparatively low and widely scattered. This is considered to be caused by distortion of the film around the tube. Therefore, the applicability of Wilke's results should be limited to the liquid films with a uniform thickness along the periphery and with small temperature difference in it.

Trommelen [8] supposed that the surface tension through its effect on the wave formation conceivably affects the heat transfer, and correlated the heat-transfer data to the Weber number $\overline{We} = \rho u_f^2 y_i/\sigma$. But it is doubtful that such a relationship involving only Weber number is accurate in accounting the heat-transfer coefficient of a non-uniform film. As Ponter and Davies [9] have pointed out, Trommelen's results for water at low Weber numbers might include data under the conditions where film breakdown had occurred. However, it is likely to be that the non-uniform film formation reduces the heat-transfer coefficient to the values lower than that determined by equations (4)–(7).

## 5. FILM BREAKDOWN

### 5.1. *On previous works*

In the previous investigations on film breakdown conditions termed as the "minimum wetting rate" of falling liquid films and the "dryout" in annular two phase flow, theoretical considerations have usually been concerned with whether a dry patch formed on the surface will remain or be re-wetted [10–17]. Therefore, a force balance at the upstream stagnation point of a dry patch has been discussed on various models of flow. However, as these models have included a contact angle of the liquid at the point of breakdown, lack of data on the contact angle [17–19], whose measurement is very difficult, has precluded any quantitative understanding of the film breakdown. An alternative approach to predict the film breakdown conditions has been conducted from a viewpoint of total mechanical (kinetic and surface) energy flow by Hartley and Murgatroyd [13] and Bankoff [19]. On the other hand, Hallett [20] correlated the minimum wetting rate during heat transfer with the difference in surface tension set up due to uneven heating caused by the wave motion of the film, and Bankoff [21] analyzed the effect of the vapour leaving normally to the film surface on the wave amplitude of a thin liquid film.

*The mechanism for breakdown of falling liquid films.* As mentioned above, this problem has been discussed from various viewpoints. However, in the liquid films of the present study, highly subcooled and with heat addition from the wall surface, the dry patch formation is considered, as shown qualitatively by Norman and McIntyre [10], to be caused by local variation of surface tension, i.e. so-called Marangoni effects due to temperature difference along the film surface. When irregularities occur in the film thickness, the region of greater thickness will have a surface of less temperature so the subsequent higher surface tension force will draw liquid from the thinner region. And finally liquid deficiency in the thinner region will occur, forming a dry patch on the wall surface.

Once a dry patch is formed, its stability is possibly discussed according to the force balance between the dynamic pressure and the surface tension at the upstream stagnation point of the dry patch. But, it should be born in mind that such a force balance is strongly affected by the degree of film distortion, because a dry patch is usually formed in the thin region of the distorted film as previously mentioned in Chapter 3.

On the other hand, in evaporating liquid films, which will be studied in Part II, the liquid surface is maintained at a temperature very close to the saturation value. Hence, the process of film breakdown in evaporating films is supposed to be much different from that in highly subcooled films. The breakdown of evaporating films may occur at high heat fluxes as a result of total loss of liquid by evaporation and droplet entrainment, as shown by Hewitt *et al.* [22] and Shires *et al.* [23]. In some particular cases, the effect of the vaporization on the wave amplitude of a liquid film may have connection with the film breakdown.

### 5.2. *Experimental results*

The data of film breakdown heat fluxes are shown in Fig. 5 for a 600 mm length tube. In this figure, experimental data for several inlet water temperatures are plotted against the film flow rate, showing a couple

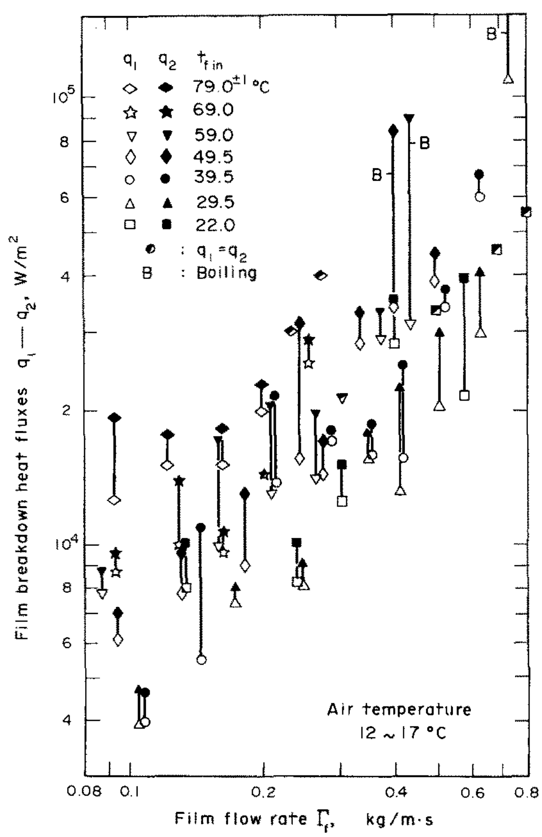


FIG. 5. Film breakdown heat fluxes ($L$ = 600 mm).

of points corresponding to the heat flux $q_1$ at which local dry patches are formed and the heat flux $q_2$ at which a dry patch becomes permanent. With increasing film flow rate and with increasing inlet water temperature, the heat flux to cause a dry patch becomes higher. The same tendency is shown in the results of Shires *et al.* [23]. In addition, the symbol $B$ in this figure shows the heat flux at which incipient surface boiling was observed in the thin region of the film.

For a 1000 mm length tube, the experimental data plotted in the way showed the same tendency as above, except that most of them showed $q_1 = q_2$.

term decreases to a value equal to the surface tension term at the stagnation point of the dry patch. In fact, a dry patch is usually formed in the thin region of the distorted film as shown in Fig. 6(b). In the thin film region, however, there remains a small wave motion which causes the dynamic pressure term of the film to fluctuate. Consequently, it seems to be that the dry patch can be re-wetted at the heat flux $q_1$ because the maximum of the dynamic pressure term is above the surface tension term; at the heat flux $q_2$ the dry patch can exist permanently because the film is further distorted and even the maximum of the pressure term is decreased below the surface tension term.

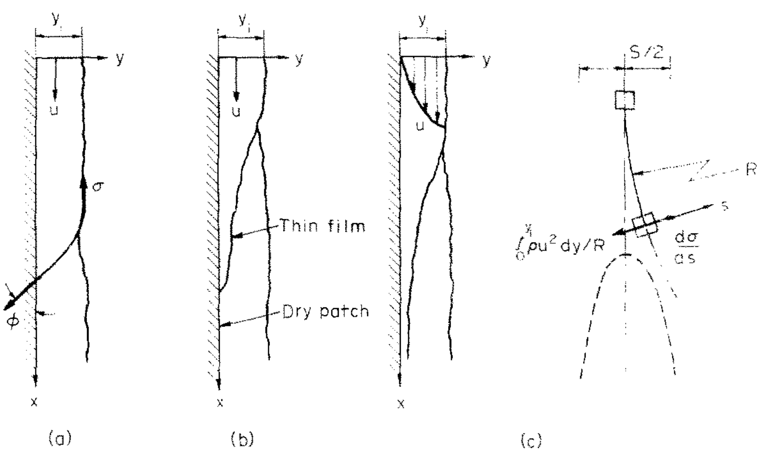


FIG. 6. Dry patch formation and film distortion.

Here, assuming the existence of a dry patch in uniform film flow as shown in Fig. 6(a), consider the force balance at the upstream stagnation point of the dry patch. Denoting the dynamic pressure of the film as $P_d$, the film thickness as $y_i$, and the surface tension neglecting its variation with temperature as $\sigma$, then

$$P_d y_i = \int_0^{y_i} \rho u^2 \, dy/2 = \sigma(1-\cos\phi)$$

or

$$We = P_d y_i/\sigma = 1-\cos\phi \tag{8}$$

where $\phi$ is the contact angle of the liquid which is determined by conditions of the heating surface, environment and fluid at their triple interface. Although the value of contact angle is not definite, the Weber number at the condition of the force balance can be estimated to be $\dot{W}e = 0.29$, by assuming the value of $\phi = 45^\circ$ given by Hartley and Murgatroyd [13]. The value of $We = 0.29$ seems to be the condition of minimum wetting rate for isothermal uniform film flow.

Film flow rates in this experiment were above this minimum wetting rate and calculated to be $We = 0.3$–$6$ by introducing the universal velocity profile. That is, the mean dynamic pressure term $P_d y_i$ of uniform films always exceeds the surface tension term $\sigma(1-\cos\phi)$. Therefore, the dry patch can not be stable unless the film upstream of the dry patch becomes thinner than the average and so the dynamic pressure

### 5.3. *Parameter for the film distortion*

The distortion of film flow is supposed to be promoted by the surface tension difference set up in the film and suppressed by the inertia force (or dynamic pressure) of the film flow. Therefore, a dimensionless parameter $K$ is introduced to evaluate the film distortion as a ratio of the surface tension difference $\Delta\sigma$ to the dynamic pressure multiplied by the film thickness of uniform film flow $P_d y_i$:

$$K = \Delta\sigma/P_d y_i = 2\Delta\sigma \Big/ \int_0^{y_i} \rho u^2 \, dy. \tag{9}$$

Although $\Delta\sigma$ is of the order smaller than $P_d y_i$, it can be large enough to deflect the film flow with an increase of heat flux.

Here, consider the film flow whose streamline curves with a radius $R$ due to the effect of $\Delta\sigma$ acting normal to the streamline and parallel to the wall surface, as shown in Fig. 6(c). The radial balance between the centrifugal force and the surface tension difference gives

$$\int_0^{y_i} \rho u^2 \, dy/R = \frac{\partial\sigma}{\partial s} \tag{10}$$

where $s$ is a radial distance measured across the streamline. Denoting the film width related with the film distortion as $S$, the surface tension difference acting on a half width of the symmetrically distorted film is given by

$$\Delta\sigma = \int_0^{S/2} \frac{\partial \sigma}{\partial s}\,\mathrm{d}s = S\int_0^{y_i} \rho u^2\,\mathrm{d}y/2R. \quad (11)$$

From equations (9) and (11), the distortion parameter can be written in the form,

$$K = S/R. \quad (12)$$

This indicates that the film flow possibly changes its direction by a local curvature $1/R$ relative to the value of $K$.

As the temperature coefficient of the surface tension of water is approximately constant at the atmospheric pressure, $\Delta\sigma$ is written by

$$\Delta\sigma \simeq \left(\frac{\partial \sigma}{\partial t}\right)\Delta t. \quad (13)$$

In this paper, the temperature difference between the bulk mean liquid and the wall surface is assumed to be applicable for $\Delta t$, as a value proportional to the maximum temperature difference allowed locally in the film. Then

$$K = \left(\frac{\partial \sigma}{\partial t}\right)(t_f - t_o)/P_d y_i. \quad (14)$$

The values of $y_i$, $P_d$, $(t_f - t_o)$ and $K$ can be calculated for both laminar and turbulent falling films by determining the velocity and temperature distributions in the films.

Denote the dimensionless parameters of velocity and distance from the wall as $u^+ = u/(\tau_o/\rho)^{1/2}$ and $y^+ = y(\tau_o/\rho)^{1/2}/\nu$. The wall shear stress is given by $\tau_o = \rho g y_i$ for falling films, then the shear stress distribution is

$$\tau/\tau_o = 1 - y/y_i = 1 - y^+/y_i^+. \quad (15)$$

The film Reynolds number and the mean dynamic pressure are expressed as

$$Re = 4\Gamma_f/\mu = 4\int_0^{y_i} \rho u\,\mathrm{d}y/\mu = 4\int_0^{y_i^+} u^+\,\mathrm{d}y^+ \quad (16)$$

$$P_d = \int_0^{y_i} \rho u^2\,\mathrm{d}y/2y_i = \tau_o \int_0^{y_i^+} (u^+)^2\,\mathrm{d}y^+/2y_i^+. \quad (17)$$

Considering a condition of fully developed flow with constant heat rate along the tube and assuming all of the heat transferred is absorbed in the liquid film, i.e. $(\partial t/\partial x) = \text{const.}$ and $q_i = 0$, the heat flux distribution is given by

$$q/q_o = 1 - \int_0^{y} u\,\mathrm{d}y \bigg/ \int_0^{y_i} u\,\mathrm{d}y = 1 - 4\int_0^{y^+} u^+\,\mathrm{d}y^+/Re. \quad (18)$$

Therefore, the velocity and temperature distributions and also the value of $K$ can be obtained by applying the above equations.

(1) *Laminar flow regime.* The shear stress and heat flux distributions for laminar flow are given as follows:

$$\tau = \mu\frac{\partial u}{\partial y}; \quad \frac{\tau}{\tau_o} = \frac{\mathrm{d}u^+}{\mathrm{d}y^+} \quad (19)$$

$$q = -k\frac{\partial t}{\partial y}; \quad \frac{q}{q_o} = \frac{1}{Pr}\frac{\mathrm{d}t^+}{\mathrm{d}y^+} \quad (20)$$

where $t^+$ is the dimensionless parameter of temperature defined by $t^+ = \rho c(t_o - t)(\tau_o/\rho)^{1/2}/q_o$. Substituting equation (15) into equation (19),

$$u^+ = y^+ - (y^+)^2/2y_i^+. \quad (21)$$

Then, from equations (16) and (17), the following expressions for the film thickness and the dynamic pressure are obtained:

$$y_i^+ = (\tfrac{3}{4}Re)^{1/2} \quad (22)$$

$$P_d/\rho g\left(\frac{\nu^2}{g}\right)^{1/3} = \tfrac{1}{15}(\tfrac{3}{4})^{4/3} Re^{4/3}. \quad (23)$$

The temperature profile is expressed from equations (18), (20) and (21) as

$$t^+/Pr = \int_0^{y^+}\left(\frac{q}{q_o}\right)\mathrm{d}y^+ = y^+ - \frac{(y^+)^3}{2(y_i^+)^2} + \frac{(y^+)^4}{8(y_i^+)^3}. \quad (24)$$

Then, the dimensionless bulk mean temperature $t_f^+ = \rho c(t_o - t_f)(\tau_o/\rho)^{1/2}/q_o$ can be determined as

$$t_f^+ = \int_0^{y_i^+} t^+ u^+\,\mathrm{d}y^+ \bigg/ \int_0^{y_i^+} u^+\,\mathrm{d}y^+ = \tfrac{17}{35} y_i^+ Pr. \quad (25)$$

So that, the temperature difference becomes

$$-\Delta t = t_o - t_f = \frac{17}{35}\frac{q_o \nu y_i^+}{\kappa(\tau_o/\rho)^{1/2}} = \frac{17}{35}\frac{q_o}{k} y_i. \quad (26)$$

Therefore, the expression for the distortion parameter of laminar falling films, which is now denoted as $K_l$, is obtained from equations (14), (23) and (26) as follows:

$$K_l = 10.7\left(-\frac{\partial \sigma}{\partial t}\right) q_o/k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Re^{4/3}. \quad (27)$$

(2) *Turbulent flow regime.* A number of investigations [2, 7] on the mean thickness of falling liquid film have shown that the measured thickness is in good agreement with that calculated by introducing the universal velocity profile in a range of Reynolds numbers from 1000 to 7000. Application of the universal velocity profile to the films leads to the following relationships of $Re - y_i^+$ and $P_d - y_i^+$ for a range of $y_i^+ \geqslant 30$:

$$Re = -256 + 12y_i^+ + 10y_i^+ \ln y_i^+ \quad (28)$$

$$P_d/\rho g\left(\frac{\nu^2}{g}\right)^{1/3} = \{-1088 + 15.3y_i^+ + 15.0y_i^+ \ln y_i^+ + 6.25y_i^+(\ln y_i^+)^2\}/2(y_i^+)^{1/3}. \quad (29)$$

In a range of Reynolds numbers from 1500 to 7000, these values of $Re$ and $P_d$ can be obtained, to a close enough approximation, by the following simple equations:

$$y_i^+ = 0.102 Re^{0.80} \quad (28)'$$

$$P_d/\rho g\left(\frac{\nu^2}{g}\right)^{1/3} = 0.80 Re^{0.92}. \quad (29)'$$

For the turbulent film flow, the measured heat-transfer coefficient showed a trend similar to that of equation (6) which was presented by Wilke for the

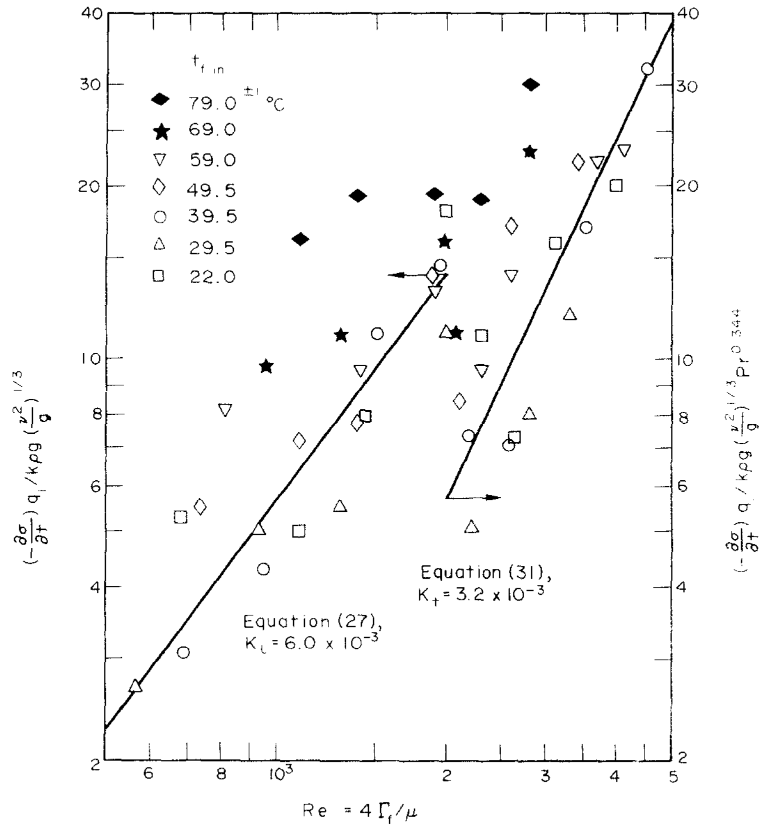


FIG. 7. Heat flux of local dry patch formation ($L = 600$ mm). (Temperature $t_f > 70°$C for data plotted by ◆★.)

films of Reynolds numbers from 1600 to 3200. So the temperature difference may be expressed from equation (6) as

$$-\Delta t = t_o - t_f = \frac{q_o}{h} = q_o\left(\frac{\nu^2}{g}\right)^{1/3} / 0.00102kPr^{0.344}Re^{2/3}. \quad (30)$$

Therefore, the expression for turbulent falling films, which is now denoted as $K_t$, is obtained as follows:

$$K_t = 5620\left(-\frac{\partial\sigma}{\partial t}\right)q_o/k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Pr^{0.344}Re^{2.12}. \quad (31)$$

Experimental results of film breakdown are correlated in dimensionless forms with the distortion parameters derived above. In the case of vertical falling films, the transition from laminar to turbulent flow is supposed to take place progressively in a range of Reynolds numbers about 1000–3500, as expected from the trend of film thickness [2] and heat transfer coefficient (Fig. 4). Here, $K_l$ for laminar flow is used to correlate the data in a range where $Re < 2000$, and $K_t$ for turbulent flow in a range where $Re > 2000$.

Figure 7 shows the results for a 600 mm length tube, plotted the dimensionless groups involving the film breakdown heat flux $q_1$, $(-\partial\sigma/\partial t)q_1/k\rho g(\nu^2/g)^{1/3}$ for $Re < 2000$, and $(-\partial\sigma/\partial t)q_1/k\rho g(\nu^2/g)^{1/3}Pr^{0.344}$ for $Re > 2000$, against the film Reynolds numbers. The physical properties used are for the bulk mean temperature of liquid at the location of film breakdown. When water is highly subcooled, $t_f < 70°$C, experimental results agree approximately with a solid line proportional to $Re^{4/3}$ for $Re < 2000$ and that to $Re^{2.12}$ for $Re > 2000$. Hence, the values of $K_l$ and $K_t$ are considered to be constants at the film breakdown heat flux $q_1$. The values of $K_l$ and $K_t$ are also found to be constants at the heat flux $q_2$, as shown in Fig. 8. These values of the distortion parameters at the film breakdown heat fluxes $q_1$ and $q_2$ obtained with 600 mm and 1000 mm length tubes are tabulated in Table 1.

Here, denoting the distortion parameters at the heat fluxes $q_1$ and $q_2$ as $K_{q1}$ and $K_{q2}$ respectively, $K_{q1}$ shows the same value regardless of the tube length. $K_{q2}$ shows a value about 30% higher than $K_{q1}$ in the 600 mm length tube, while in the 1000 mm length tube, $K_{q2}$ shows a value almost equal to $K_{q1}$. Therefore, $K_{q1}$ is estimated to be a constant in any length of tube longer than 600 mm at the least, while $K_{q2}$ possibly approaches to $K_{q1}$ as the tube becomes longer.

All of the experimental data of film breakdown measured for highly subcooled water below 70°C are then correlated by the following equations.

Table 1. Parameters $K_l$ and $K_t$ at film breakdown conditions

| Tube length | 600 mm | | 1000 mm | |
|---|---|---|---|---|
| Heat flux | $q_1$ | $q_2$ | $q_1$ | $q_2$ |
| $K_l \times 10^3$ | 6.0 | 8.0 | 6.0 | 6.0 |
| $K_t \times 10^3$ | 3.2 | 4.2 | 3.2 | 3.2 |

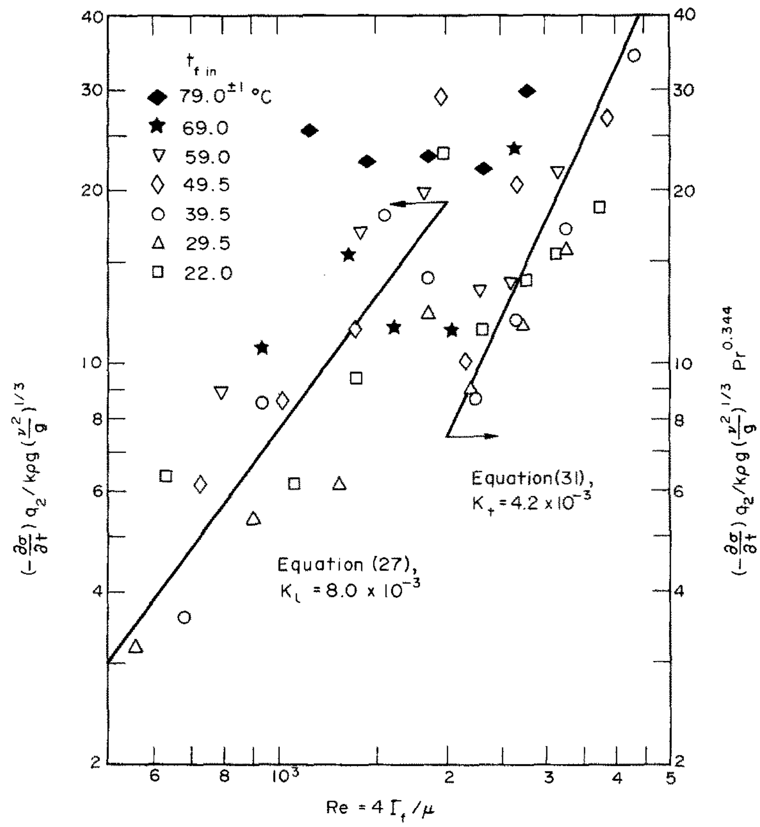


FIG. 8. Heat flux of permanent dry patch formation ($L$ = 600 mm). (Temperature $t_f$ > 70°C for data plotted by ◆★.)

In a 600 mm length tube,
for laminar regime;

$$q_1 = 5.6 \times 10^{-4} k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Re^{4/3} \Big/ \left(-\frac{\partial\sigma}{\partial t}\right) \quad (32)$$

$$q_2 = 7.5 \times 10^{-4} k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Re^{4/3} \Big/ \left(-\frac{\partial\sigma}{\partial t}\right) \quad (33)$$

for turbulent regime;

$$q_1 = 5.7 \times 10^{-7} k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Pr^{0.344} Re^{2.12} \Big/ \left(-\frac{\partial\sigma}{\partial t}\right) \quad (34)$$

$$q_2 = 7.5 \times 10^{-7} k\rho g\left(\frac{\nu^2}{g}\right)^{1/3} Pr^{0.344} Re^{2.12} \Big/ \left(-\frac{\partial\sigma}{\partial t}\right). \quad (35)$$

In a 1000 mm length tube, $q_1$ and $q_2$ are equal to each other and given by equations (32) and (34) for laminar and turbulent regimes respectively.

### 5.4. *Discussion of the distorted film flow*

In the previous section of this Chapter, it was described that when the parameter $K$ attained a constant value by increasing the heat flux, even the film flow with a comparatively high Weber number came to be distorted along the tube periphery, and a dry patch was formed in the thin region of the distorted film. In the present context, some characteristics of the distorted film are discussed for the laminar film flow.

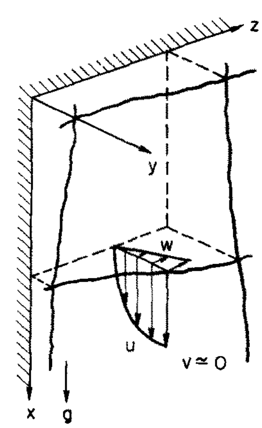


FIG. 9. Distorted film flow.

Consider the case where a non-uniform film flows down on a vertical heating wall in steady-state, as sketched together with the coordinate system in Fig. 9. It is assumed that acceleration within the film can be ignored, and that a velocity component normal to the wall is much smaller than the others parallel to the wall and can be neglected. Accordingly, the equations of motion are simplified to

$$\mu\frac{\partial^2 u}{\partial y^2} + \rho g = 0, \quad \mu\frac{\partial^2 w}{\partial y^2} = 0. \quad (36)$$

Denoting the surface tension of the liquid as $\sigma$, the boundary conditions at the wall and the film surface are given by

$$y=0;\quad u=0,\ w=0$$

$$y=y_i;\quad \mu\frac{\partial u}{\partial y}\bigg|_{y=y_i}=\frac{\partial\sigma}{\partial x},\quad \mu\frac{\partial w}{\partial y}\bigg|_{y=y_i}=\frac{\partial\sigma}{\partial z}. \tag{37}$$

Assuming that physical properties of the liquid except for surface tension are constants and integrating equation (36) with above boundary conditions, the velocities $u$ and $w$ within the film are solved. Then the mean velocities $u_f$ and $w_f$ are obtained by integrating each of the velocity profiles across the film;

$$\left.\begin{aligned} u_f &= \int_0^{y_i} u\,\mathrm{d}y/y_i = \frac{\rho g}{3\mu}y_i^2+\frac{1}{2\mu}\frac{\partial\sigma}{\partial x}y_i \\ w_f &= \int_0^{y_i} w\,\mathrm{d}y/y_i = \frac{1}{2\mu}\frac{\partial\sigma}{\partial z}y_i. \end{aligned}\right\} \tag{38}$$

The equation of continuity in flow rate is

$$\frac{\partial}{\partial x}(\rho u_f y_i)+\frac{\partial}{\partial z}(\rho w_f y_i)=0. \tag{39}$$

Considering the case where heat flux is uniform over the wall and using the bulk mean temperature of the liquid film $t_f$, the equation of energy transfer is written as

$$\frac{\partial}{\partial x}(\rho c u_f t_f y_i)+\frac{\partial}{\partial z}(\rho c w_f t_f y_i)=q_o \tag{40}$$

From this and equation (39),

$$\rho c u_f y_i\frac{\partial t_f}{\partial x}+\rho c w_f y_i\frac{\partial t_f}{\partial z}=q_o. \tag{41}$$

A simplification of the problem can be made by assuming that the temperature of the film surface approximates to that of the bulk mean $t_f$ and by taking into account that the temperature coefficient of surface tension of water is almost constant at the atmospheric pressure. Then

$$\frac{\partial t_f}{\partial x}\simeq\frac{\partial\sigma}{\partial x}\bigg/\frac{\partial\sigma}{\partial t},\quad \frac{\partial t_f}{\partial z}\simeq\frac{\partial\sigma}{\partial z}\bigg/\frac{\partial\sigma}{\partial t}. \tag{42}$$

Substituting this into equation (41),

$$u_f y_i\frac{\partial\sigma}{\partial x}+w_f y_i\frac{\partial\sigma}{\partial z}=\left(\frac{\partial\sigma}{\partial t}\right)q_o/\rho c$$

and substituting equation (38),

$$\left(\frac{\partial\sigma}{\partial x}\bigg/\frac{1}{3}\rho g y_i+1\right)^2+\left(\frac{\partial\sigma}{\partial z}\bigg/\frac{1}{3}\rho g y_i\right)^2=1-Q/y_i^4 \tag{43}$$

where

$$Q=18\mu\left(-\frac{\partial\sigma}{\partial t}\right)q_o/(\rho g)^2\rho c.$$

When a non-uniform film flows down on a vertical heating wall, all of the values of the surface tension gradients $\partial\sigma/\partial x$ and $\partial\sigma/\partial z$ and the film thickness $y_i$ must satisfy the relation given by equation (43). As the LHS of this equation is not less than zero, the film thickness can not be lower than a value of $Q^{1/4}$ in any part of the film. Now assuming the existence of this critical flow with the irreducible film thickness of $y_i = Q^{1/4}$ and representing it with subscript $c$, the following expressions for this flow are obtained:

$$\left(\frac{\partial\sigma}{\partial x}\right)_c=-\frac{1}{3}\rho g y_{ic},\quad \left(\frac{\partial\sigma}{\partial z}\right)_c=0$$

$$u_{fc}=\frac{1}{6}\frac{\rho g}{\mu}y_{ic}^2,\quad w_{fc}=0 \tag{44}$$

$$P_{dc}=\frac{17}{1080}\rho g\frac{g}{v^2}y_{ic}^4=\frac{17}{60}\left(-\frac{\partial\sigma}{\partial t}\right)q_o/Prk.$$

Then the mean dynamic pressure of this critical flow is expressed as follows, comparing with those of uniform flow, equations (23) and (27):

$$\begin{aligned} P_{dc}/P_d &= \frac{17}{60}\left(-\frac{\partial\sigma}{\partial t}\right)q_o \bigg/ \frac{1}{15}\left(\frac{3}{4}\right)^{4/3}Prk\rho g\left(\frac{v^2}{g}\right)^{1/3}Re^{4/3} \\ &= 0.58K_t/Pr. \end{aligned} \tag{45}$$

As the Prandtl numbers under investigation ranged from 2 to 7, $P_{dc}/P_d \simeq (0.08 \sim 0.3)K_t$.

Here, applying the same reasoning as described in Section 5.3 to the critical flow, one can evaluate the film distortion parameter $K_{tc} = \Delta\sigma_c/P_{dc}y_{ic}$. The surface tension difference allowed in the film is approximately proportional to the film thickness for a given value of heat flux, i.e.

$$\Delta\sigma/y_i \simeq (\Delta\sigma/y_i)_c$$

and thus

$$K_{tc}\simeq K_t P_d/P_{dc}=1/0.08\sim 1/0.3>1. \tag{46}$$

Consequently, the surface tension difference can be over the dynamic pressure in the critical flow. This suggests that the surface tension difference may cause the film breakdown, if the film is disturbed by the influence of the wave motion on the thick film surrounding the thin film region. Once a dry patch is formed on the wall, the force sustaining the film at the top end boundary of the dry patch depends on that of the absolute surface tension, which is much greater than that of the differential surface tension, i.e.

$$\Delta\sigma<\sigma(1-\cos\phi).$$

Therefore, the dry patch area extends not only downstream but also upstream, and a final force balance is attained at the top end boundary of the dry patch in contact with a rather thick film.

Although calculations made above are based on several rough assumptions, the critical film thickness $y_{ic} = Q^{1/4}$ can be estimated for the film breakdown heat flux $q_1$ given by equation (32), using the $y_i - Re$ relationship of equation (22), as

$$\begin{aligned} y_{ic} &= \left\{18\mu\left(-\frac{\partial\sigma}{\partial t}\right)q_1/(\rho g)^2\rho c\right\}^{1/4} \\ &= 0.35y_i/Pr^{1/4}. \end{aligned}$$

As the range of Prandtl numbers under investigation is from 2 to 7, $y_{ic}/y_i = 0.3 \sim 0.2$. Consequently, the critical film thickness is estimated to be about 1/4 of the mean film thickness.

## 6. CONCLUSIONS

From the experimental results and discussion on heat-transfer coefficient and film breakdown conditions for subcooled water films flowing downwards on outside of a vertical tube uniformly heated, the following conclusions are obtained.

(1) In a low heat flux range, the heat-transfer coefficient to a liquid film shows a trend similar to the Wilke's value, while in a high heat flux range, near the film breakdown condition, it shows a value lower than Wilke's due to the non-uniformity of the liquid film flow.

(2) When the parameter $K$, which is a ratio of the allowable difference in surface tension to the dynamic pressure of uniform film flow, attains a constant value by increasing the heat flux, the film comes to be extremely distorted along the tube periphery and a dry patch is formed in the thin region of the distorted film. The value of $K$ at the film breakdown heat fluxes such as $q_1$ (local dry patch formation) and $q_2$ (permanent dry patch condition) can be calculated from equations (27) and (31), which are of the order of $10^{-2}$ to $10^{-3}$ as shown in Table 1.

(3) The measured values of $q_1$ are nearly equal in the two test tubes ($L = 600$ and 1000 mm), and $q_2$ approaches to $q_1$ in the long tube.

(4) On a distorted laminar film flowing down on a vertical heating surface in steady-state, a simple analysis gives the minimum thickness allowed in the film as $y_{ic} = Q^{1/4}$, where $Q = 18\mu(-\partial\sigma/\partial t)q_o/(\rho g)^2\rho c$. For this thinnest film with the thickness $y_{ic}$, $K_{lc} > 1$ is derived by calculating the dynamic pressure $P_{dc}$, which suggests that the surface tension difference can be large enough to cause film breakdown.

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## TRANSFERT THERMIQUE POUR DES FILMS TOMBANT ET RUPTURE DE FILM—I. FILMS LIQUIDES SOUS-REFROIDIS

**Résumé**—Des expériences sur le coefficient de transfert et sur les flux thermique de rupture de film sont faites pour des films d'eau sous-refroidie qui tembent le long de la surface externe d'un tube vertical uniformément chauffé.

On établit pour l'écoulement laminaire ou turbulent un paramètre de distorsion du film qui est le rapport de la différence possible en tension superficielle à la pression dynamique. Les résultats expérimentaux montrent que lorsque le paramètre atteint une valeur constante par accroissement du flux thermique, la rupture du film apparaît dans une petite région du film distordu. On présente une analyse simple pour évaluer la distorsion.

## WÄRMEÜBERGANG AN RIESELFILME UND AUFREIßEN DES FILMES TEIL 1: UNTERKÜHLTE FLÜSSIGKEITSFILME

**Zusammenfassung** —Es wird über Versuche zur Ermittlung des Wärmeübergangskoeffizienten und der Wärmestromdichte beim Aufreißen eines unterkühlten Wasserfilmes, der an der Außenseite eines gleichmässig beheizten, vertikalen Rohres abwärts fließt, berichtet. Ein Deformations-Parameter, der aus dem Verhältnis einer erlaubten Differenz der Oberflächenspannung und dem dynamischen Druck einer gleichmässigen Filmströmung gebildet wird, wird für laminare und turbulente Filmströmung abgeleitet. Die experimentellen Ergebnisse zeigen, daß das Aufreißen des Filmes sich auf einen dünnen Bereich des gestörten Filmes beschränkt, wenn der Parameter einen konstanten Wert bei wachsender Wärmestrom-erreicht. Es wird eine einfache Methode zur Abschätzung der Filmdeformation angegeben.

## ПЕРЕНОС ТЕПЛА К СТЕКАЮЩИМ ПЛЕНКАМ ЖИДКОСТИ И РАЗРУШЕНИЕ ПЛЕНКИ — 1. ПЛЕНОЧНОЕ ТЕЧЕНИЕ НЕДОГРЕТОЙ ЖИДКОСТИ

**Аннотация** — Представлено экспериментальное исследование коэффициента теплообмена и критического теплового потока при течении пленки недогретой жидкости вниз по внешней поверхности вертикальной равномерно нагреваемой трубы. Выведен параметр, характеризующий деформацию плёнки как при ламинарном, так и турбулентном течении жидкости, который представляет собой соотношение допустимой разности поверхностного натяжения к динамическому давлению равномерно стекающей пленки. Экспериментальные данные показывают, что при достижении параметром постоянного значения за счет увеличения теплового потока разрушается только очень тонкий слой деформированной пленки. Приводится простой анализ оценки процесса деформации пленки.